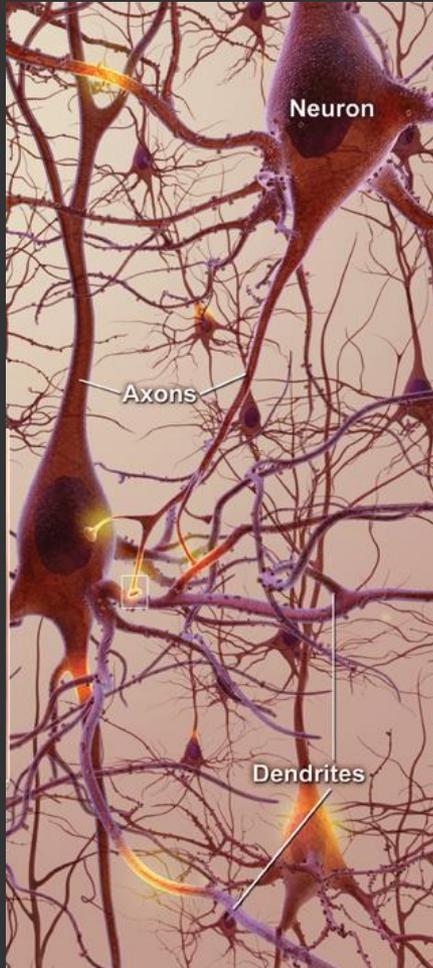


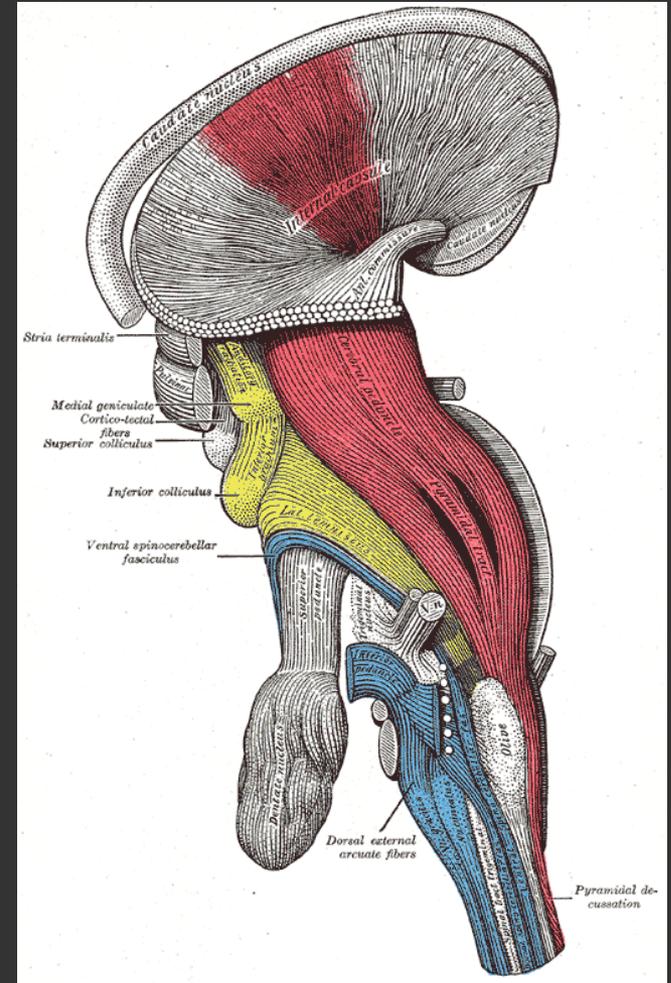
Introduction to diffusion MRI

White-matter imaging



From the National Institute on Aging

- Axons measure $\sim\mu\text{m}$ in width
- They group together in bundles that traverse the white matter
- We cannot image individual axons but we can image bundles with diffusion MRI
- Useful in studying neurodegenerative diseases, stroke, aging, development...

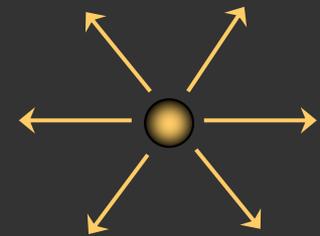


From Gray's Anatomy: IX. Neurology

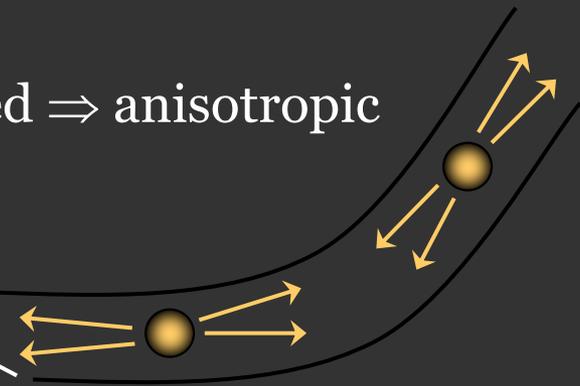
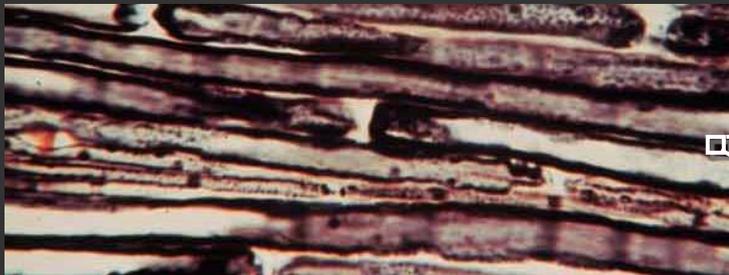
Diffusion in brain tissue

- Differentiate between tissues based on the diffusion (random motion) of water molecules within them

- Gray matter: Diffusion is unrestricted \Rightarrow isotropic

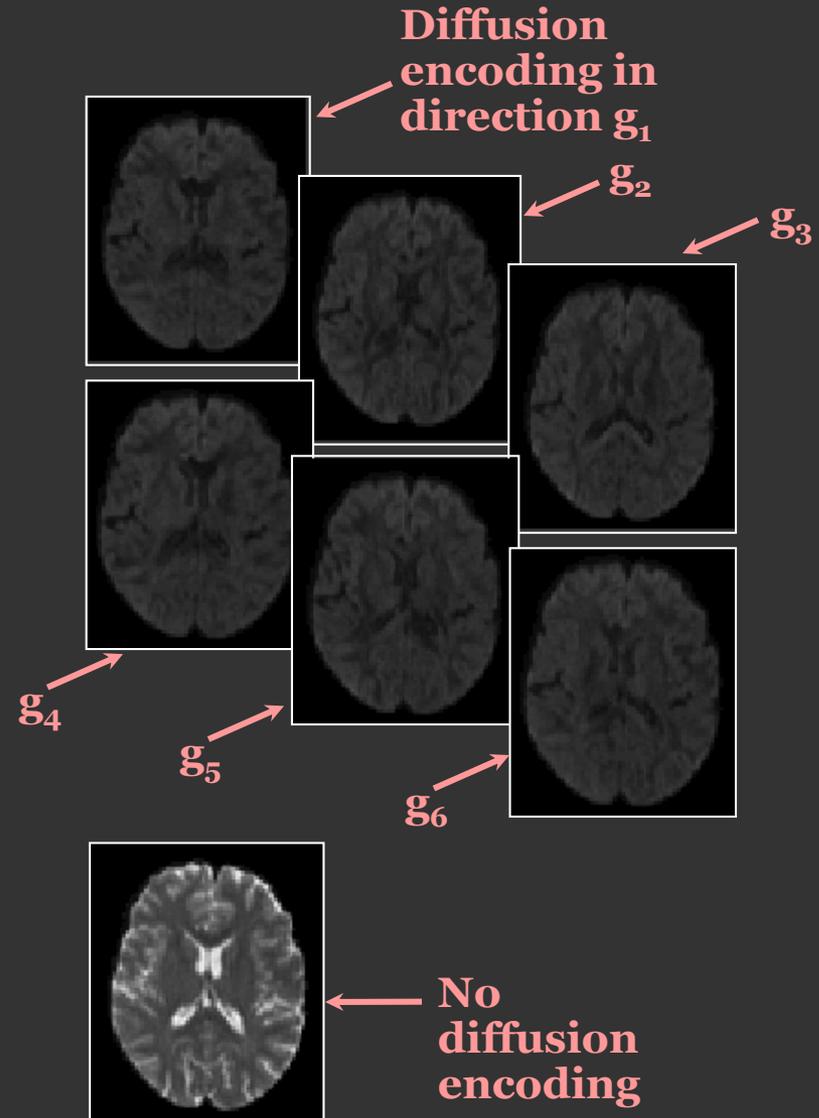


- White matter: Diffusion is restricted \Rightarrow anisotropic

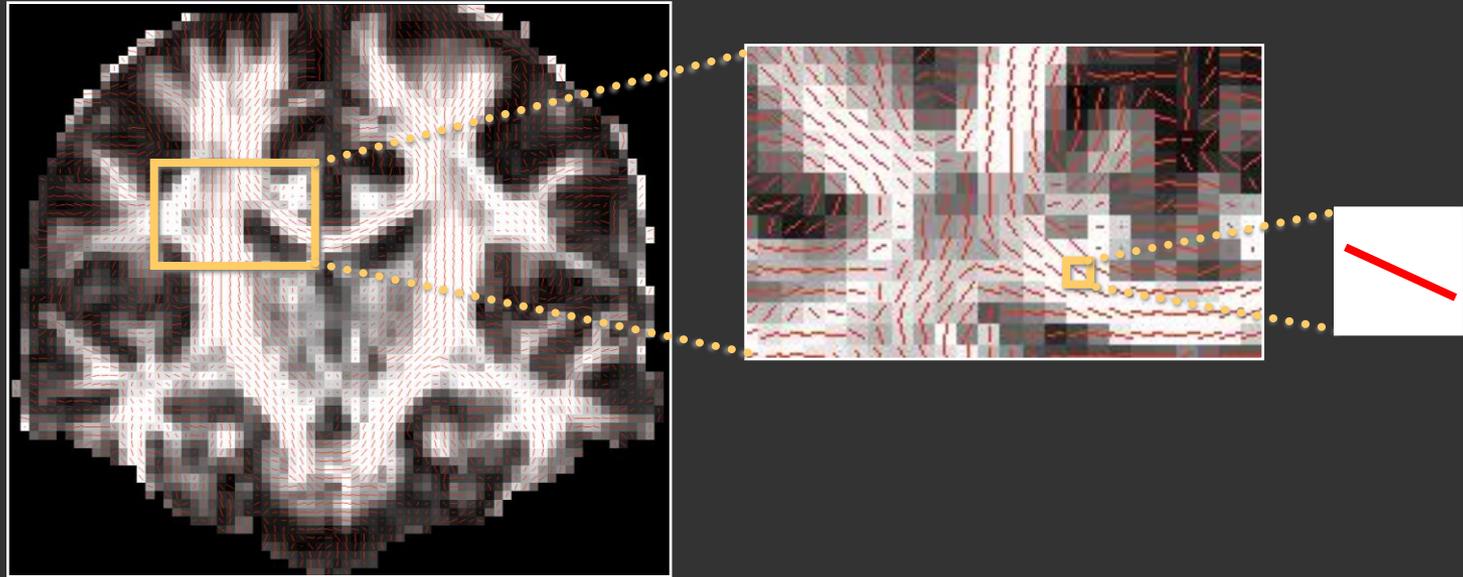


Diffusion MRI

- Magnetic resonance imaging can provide “diffusion encoding”
- Magnetic field strength is varied by gradients in different directions
- Image intensity is attenuated depending on water diffusion in each direction
- Compare with baseline images to infer on diffusion process



How to represent diffusion



- At every voxel we want to know:
 - Is this in white matter?
 - If yes, what pathway(s) is it part of?
 - What is the orientation of diffusion?
 - What is the magnitude of diffusion?
- A grayscale image cannot capture all this!

Tensors

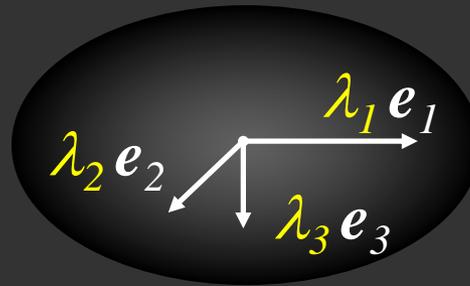
- One way to express the notion of direction is a **tensor D**
- A tensor is a 3x3 symmetric, positive-definite matrix:

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix}$$

- D is symmetric 3x3 \Rightarrow It has 6 unique elements
- Suffices to estimate the upper (lower) triangular part

Eigenvalues & eigenvectors

- The matrix D is positive-definite \Rightarrow
 - It has 3 real, positive eigenvalues $\lambda_1, \lambda_2, \lambda_3 > 0$.
 - It has 3 orthogonal eigenvectors e_1, e_2, e_3 .



$$D = \lambda_1 e_1 \cdot e_1 + \lambda_2 e_2 \cdot e_2 + \lambda_3 e_3 \cdot e_3$$

eigenvalue eigenvector

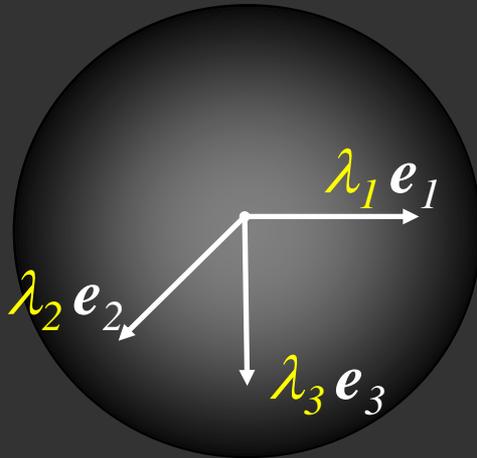
$$e_i = \begin{bmatrix} e_{ix} \\ e_{iy} \\ e_{iz} \end{bmatrix}$$

Physical interpretation

- Eigenvectors express diffusion direction
- Eigenvalues express diffusion magnitude

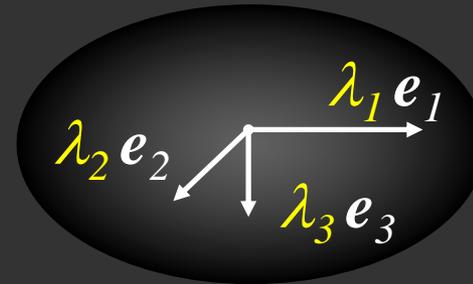
Isotropic diffusion:

$$\lambda_1 \approx \lambda_2 \approx \lambda_3$$



Anisotropic diffusion:

$$\lambda_1 \gg \lambda_2 \approx \lambda_3$$

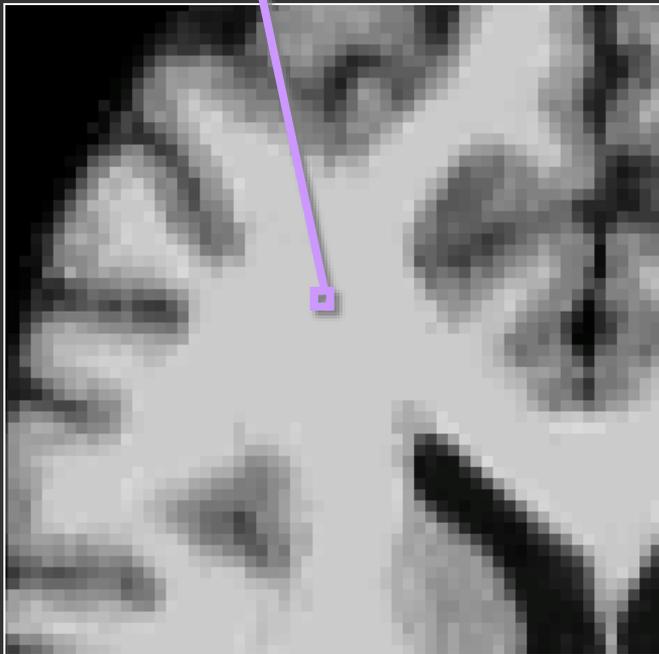


- One such ellipsoid at each voxel: Likelihood of water molecule displacements at that voxel

Diffusion tensor imaging (DTI)

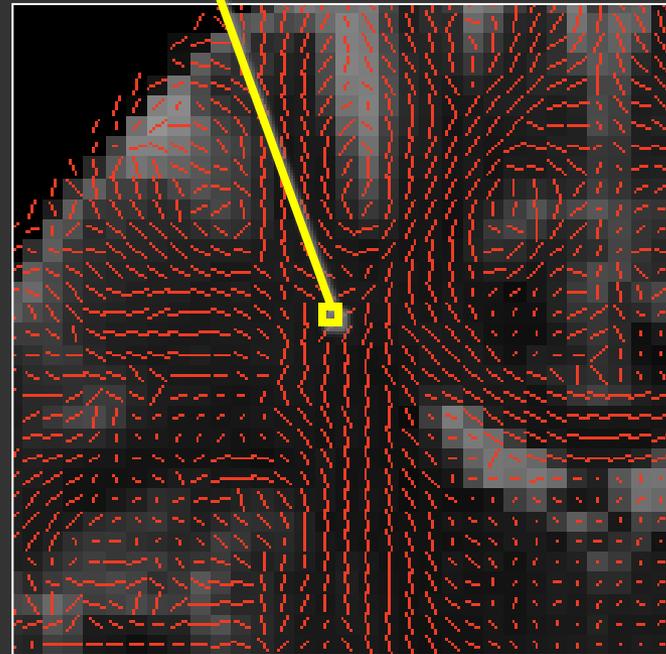
Image:

An intensity value at each voxel



Tensor map:

A tensor at each voxel

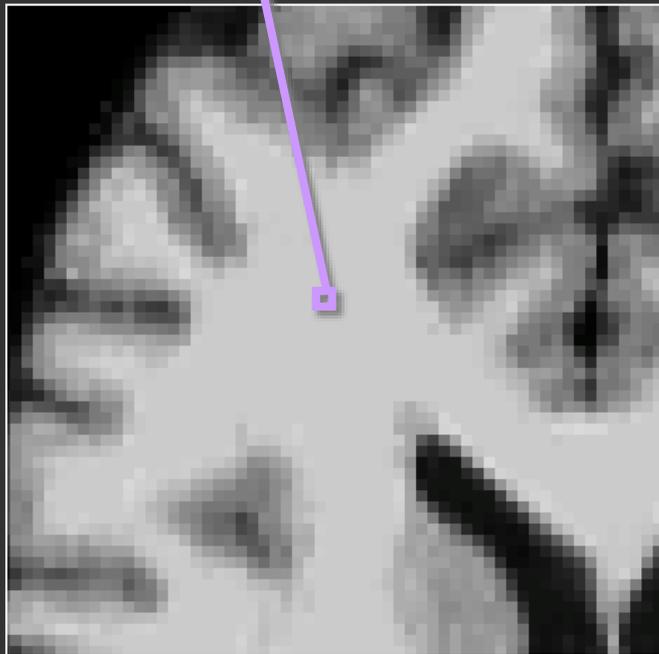


Direction of eigenvector corresponding to greatest eigenvalue

Diffusion tensor imaging (DTI)

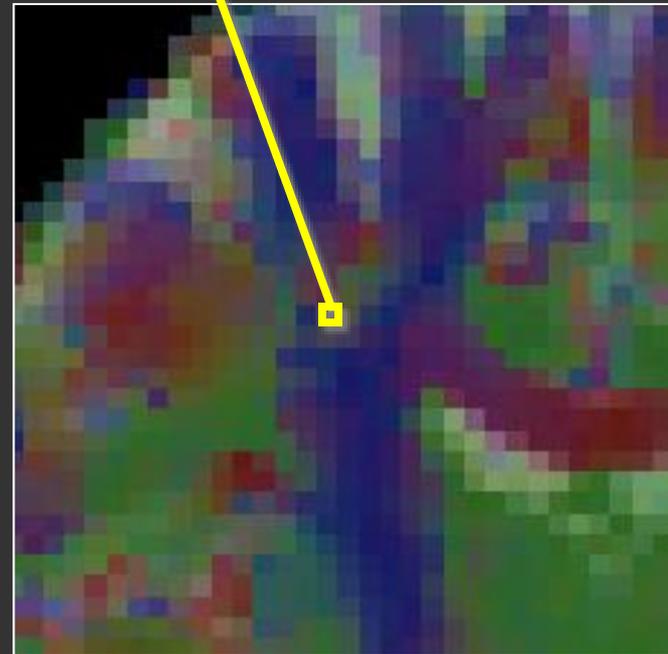
Image:

An intensity value at each voxel



Tensor map:

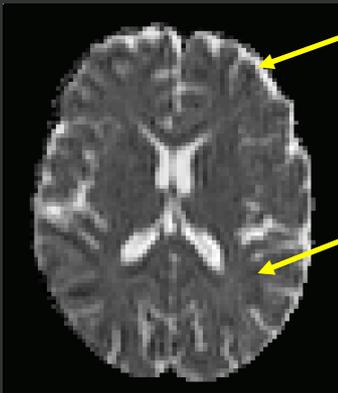
A tensor at each voxel



Direction of eigenvector corresponding to greatest eigenvalue

Red: L-R, Green: A-P, Blue: I-S

Summary measures

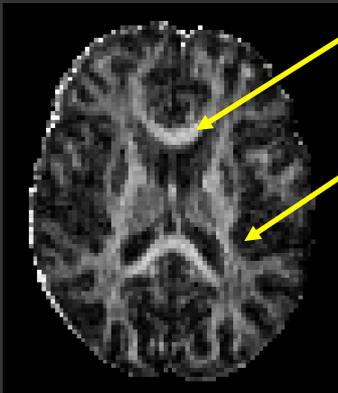


Faster diffusion

Slower diffusion

- Mean diffusivity (MD):
Mean of the 3 eigenvalues

$$MD(j) = [\lambda_1(j) + \lambda_2(j) + \lambda_3(j)]/3$$



Anisotropic diffusion

Isotropic diffusion

- Fractional anisotropy (FA):
Variance of the 3 eigenvalues,
normalized so that $0 \leq (FA) \leq 1$

$$FA(j)^2 = \frac{3}{2} \frac{[\lambda_1(j) - MD(j)]^2 + [\lambda_2(j) - MD(j)]^2 + [\lambda_3(j) - MD(j)]^2}{\lambda_1(j)^2 + \lambda_2(j)^2 + \lambda_3(j)^2}$$

More summary measures

- Axial diffusivity: Greatest of the 3 eigenvalues

$$AD(j) = \lambda_1(j)$$

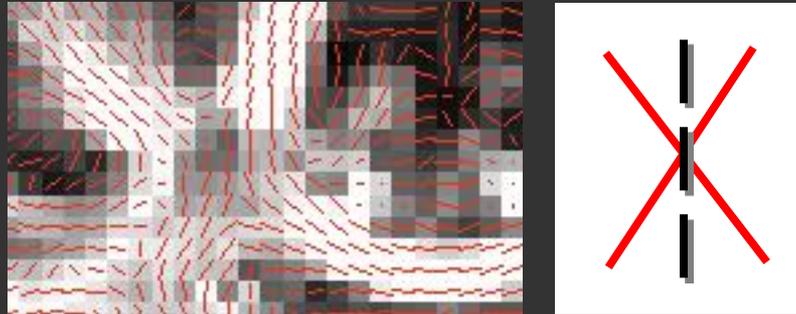
- Radial diffusivity: Average of 2 lesser eigenvalues

$$RD(j) = [\lambda_2(j) + \lambda_3(j)]/2$$

- Inter-voxel coherence: Average angle b/w the major eigenvector at some voxel and the major eigenvector at the voxels around it

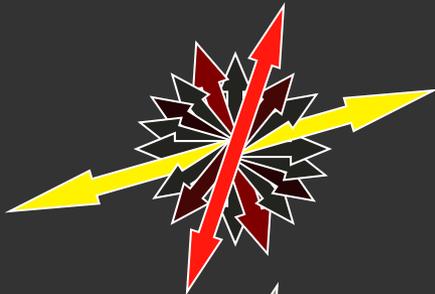
Beyond the tensor

- The tensor is an imperfect model: What if more than one major diffusion direction in the same voxel?

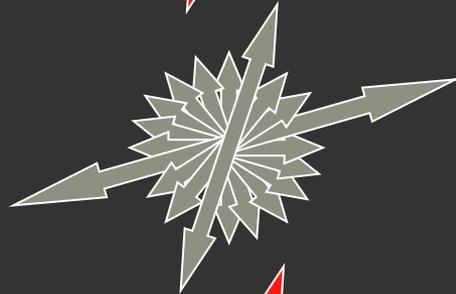


- High angular resolution diffusion imaging (HARDI): More complex models to capture more complex microarchitecture
 - Mixture of tensors [Tuch'02]
 - Higher-rank tensor [Frank'02, Özarslan'03]
 - Ball-and-stick [Behrens'03]
 - Orientation distribution function [Tuch'04]
 - Diffusion spectrum [Wedeen'05]

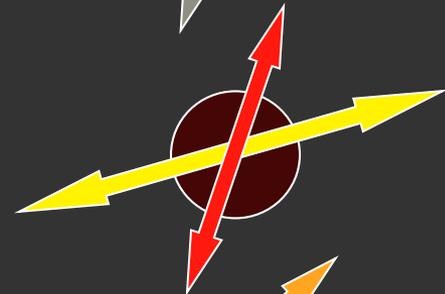
Models of diffusion



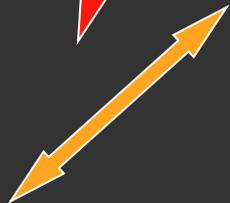
Diffusion spectrum (DSI):
Full distribution of orientation and magnitude



Orientation distribution function (Q-ball):
No magnitude info, only orientation

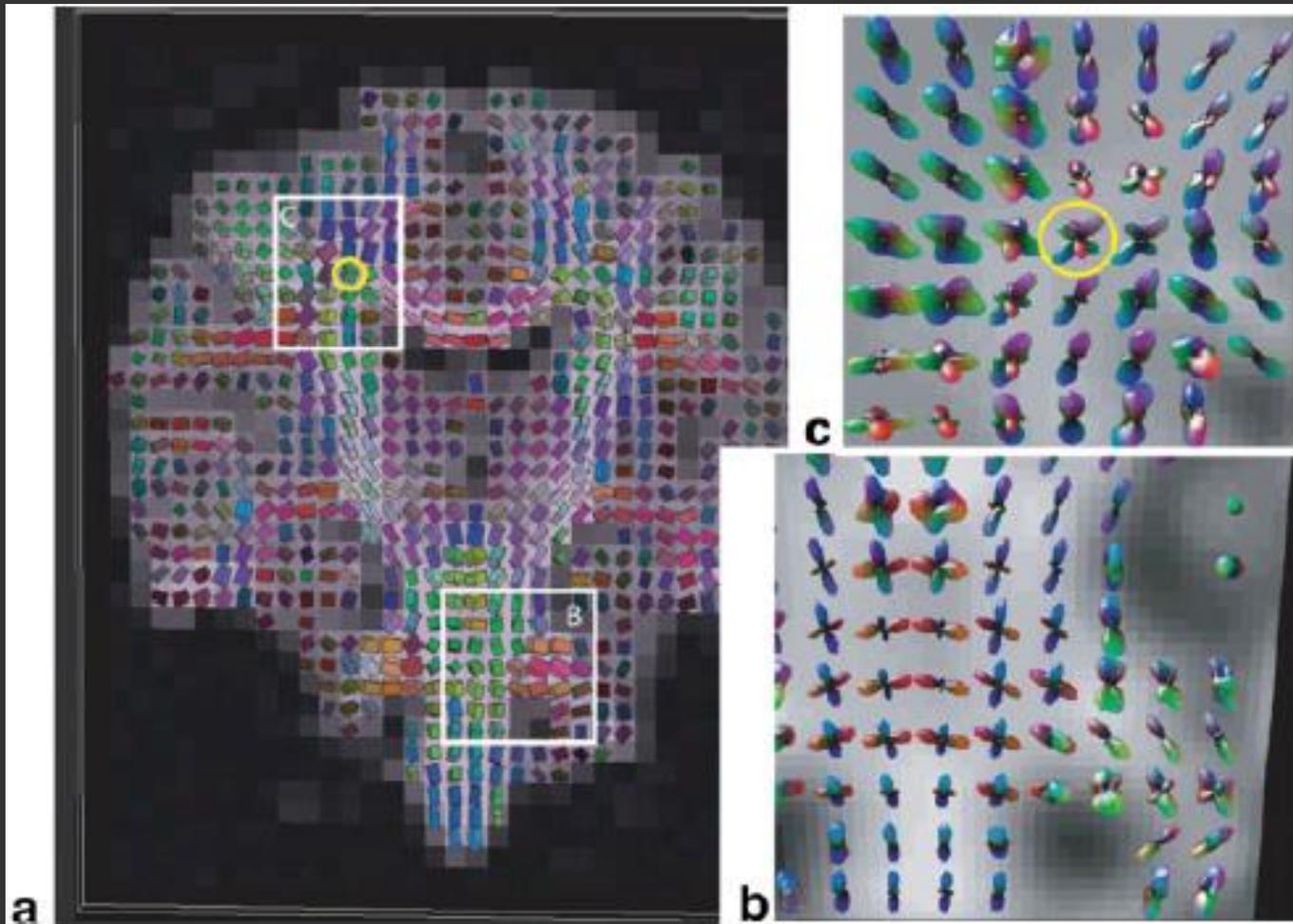


Ball-and-stick:
Orientation and magnitude for up to N anisotropic compartments



Tensor (DTI):
Single orientation and magnitude

Example: DTI vs. DSI



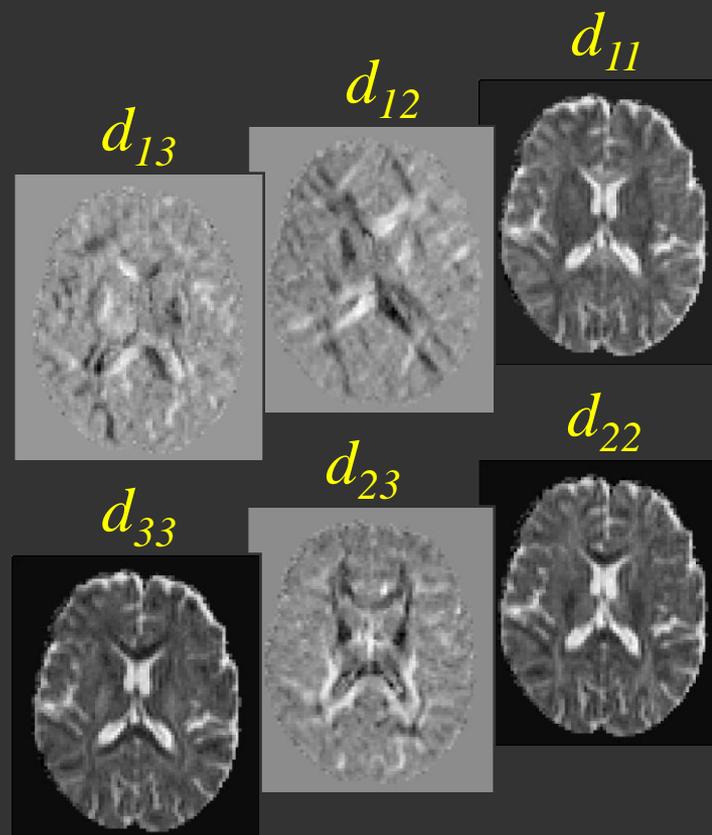
From Wedeen *et al.*, Mapping complex tissue architecture with diffusion spectrum magnetic resonance imaging, MRM 2005

Data acquisition

- Remember: A tensor has six unique parameters

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix}$$

- To estimate six parameters at each voxel, must acquire at least six diffusion-weighted images
- HARDI models have more parameters per voxel, so more images must be acquired

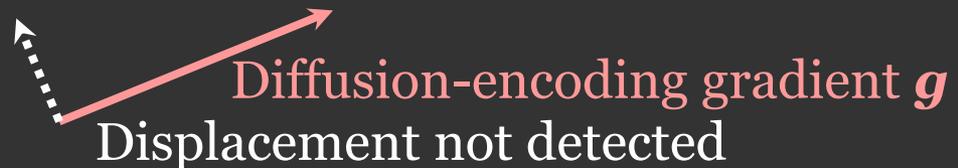


Choice 1: Gradient directions

- True diffusion direction \parallel Applied gradient direction
 \Rightarrow Maximum attenuation



- True diffusion direction \perp Applied gradient direction
 \Rightarrow No attenuation



- To capture all diffusion directions well, gradient directions should cover 3D space uniformly



How many directions?

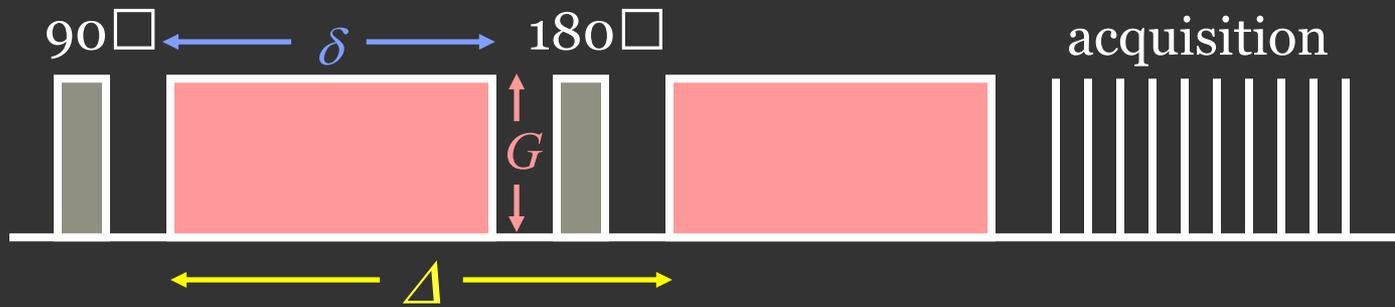
- Acquiring data with more gradient directions leads to:
 - + More reliable estimation of diffusion measures
 - Increased imaging time \Rightarrow Subject discomfort, more susceptible to artifacts due to motion, respiration, etc.
- DTI:
 - Six directions is the minimum
 - Usually a few 10's of directions
 - Diminishing returns after a certain number [Jones, 2004]
- HARDI/DSI:
 - Usually a few 100's of directions

Choice 2: The b-value

- The b-value depends on acquisition parameters:

$$b = \gamma^2 G^2 \delta^2 (\Delta - \delta/3)$$

- γ the gyromagnetic ratio
- G the strength of the diffusion-encoding gradient
- δ the duration of each diffusion-encoding pulse
- Δ the interval b/w diffusion-encoding pulses



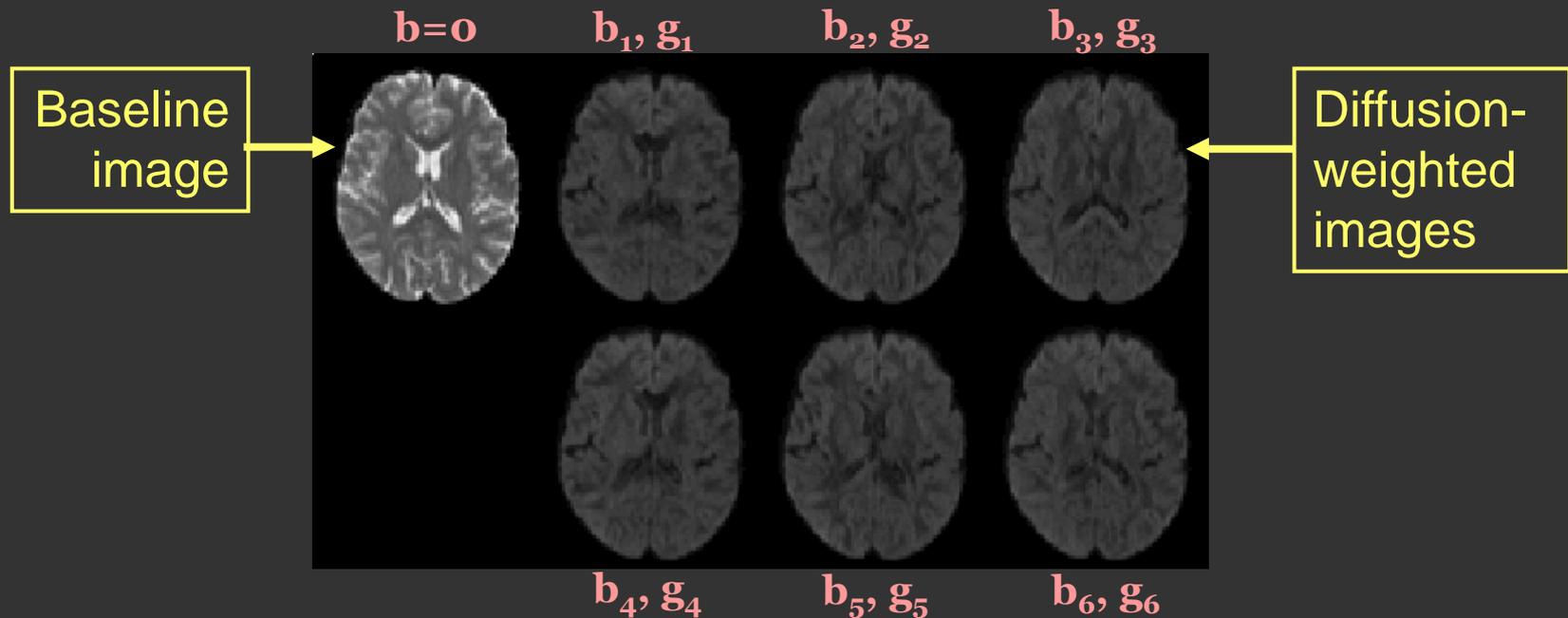
How high b-value?

- Increasing the b-value leads to:
 - + Increased contrast b/w areas of higher and lower diffusivity in principle
 - Decreased signal-to-noise ratio \Rightarrow Less reliable estimation of diffusion measures in practice
- DTI: $b \sim 1000 \text{ sec/mm}^2$
- HARDI/DSI: $b \sim 10,000 \text{ sec/mm}^2$
- Data can be acquired at multiple b-values for trade-off
- Repeat acquisition and average to increase signal-to-noise ratio

Looking at the data

A diffusion data set consists of:

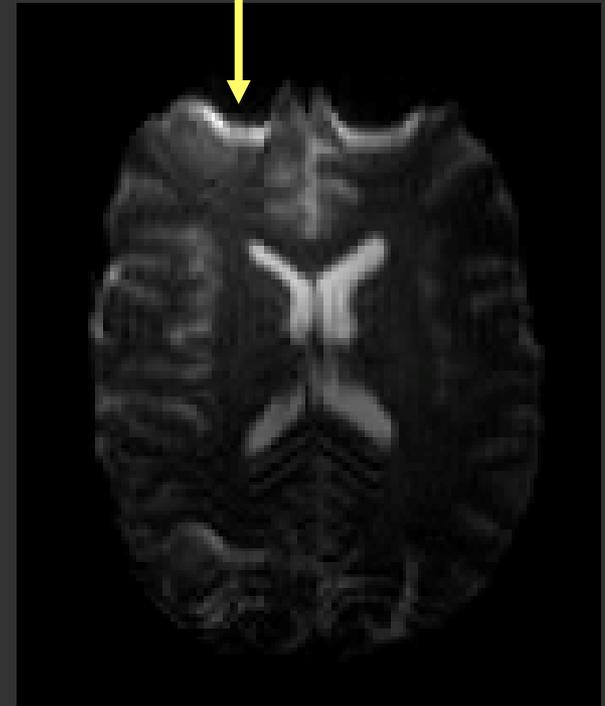
- A set of non-diffusion-weighted a.k.a “baseline” a.k.a. “low-b” images ($b\text{-value} = 0$)
- A set of diffusion-weighted (DW) images acquired with different gradient directions $\mathbf{g}_1, \mathbf{g}_2, \dots$ and $b\text{-value} > 0$
- The diffusion-weighted images have lower intensity values



Distortions: Field inhomogeneities

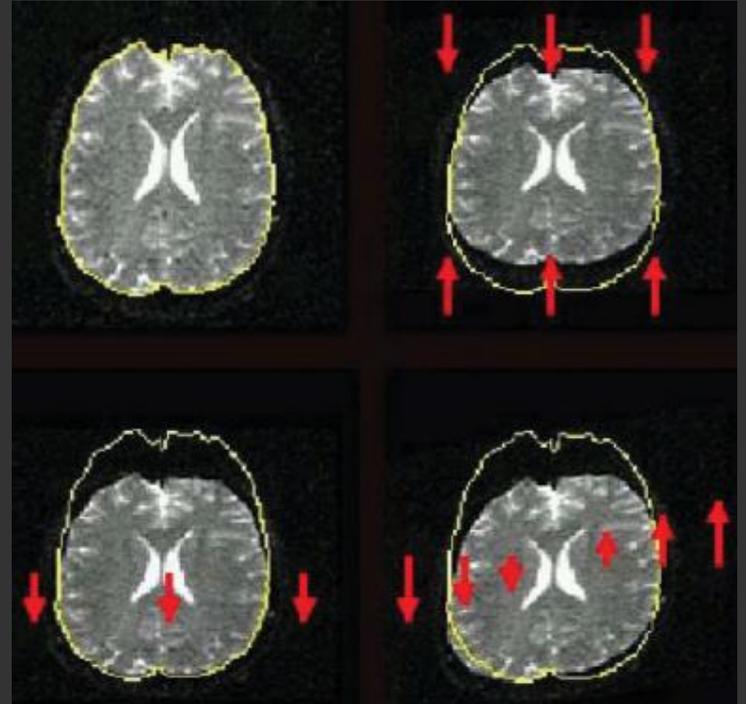
- Causes:
 - **Scanner-dependent** (imperfections of main magnetic field)
 - **Subject-dependent** (changes in magnetic susceptibility in tissue/air interfaces)
- Results:
 - Signal loss in interface areas
 - Geometric distortions (warping) of the entire image

Signal loss



Distortions: Eddy currents

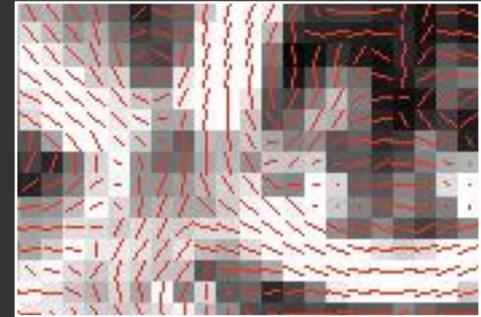
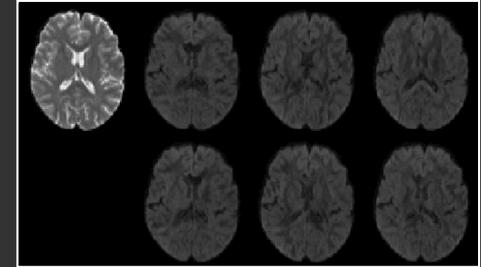
- Cause: Fast switching of diffusion-encoding gradients induces eddy currents in conducting components
- Eddy currents lead to residual gradients that shift the diffusion gradients
- The shifts are **direction-dependent**, *i.e.*, different for each DW image
- Result: Geometric distortions



From Le Bihan *et al.*, Artifacts and pitfalls in diffusion MRI, JMIR 2006

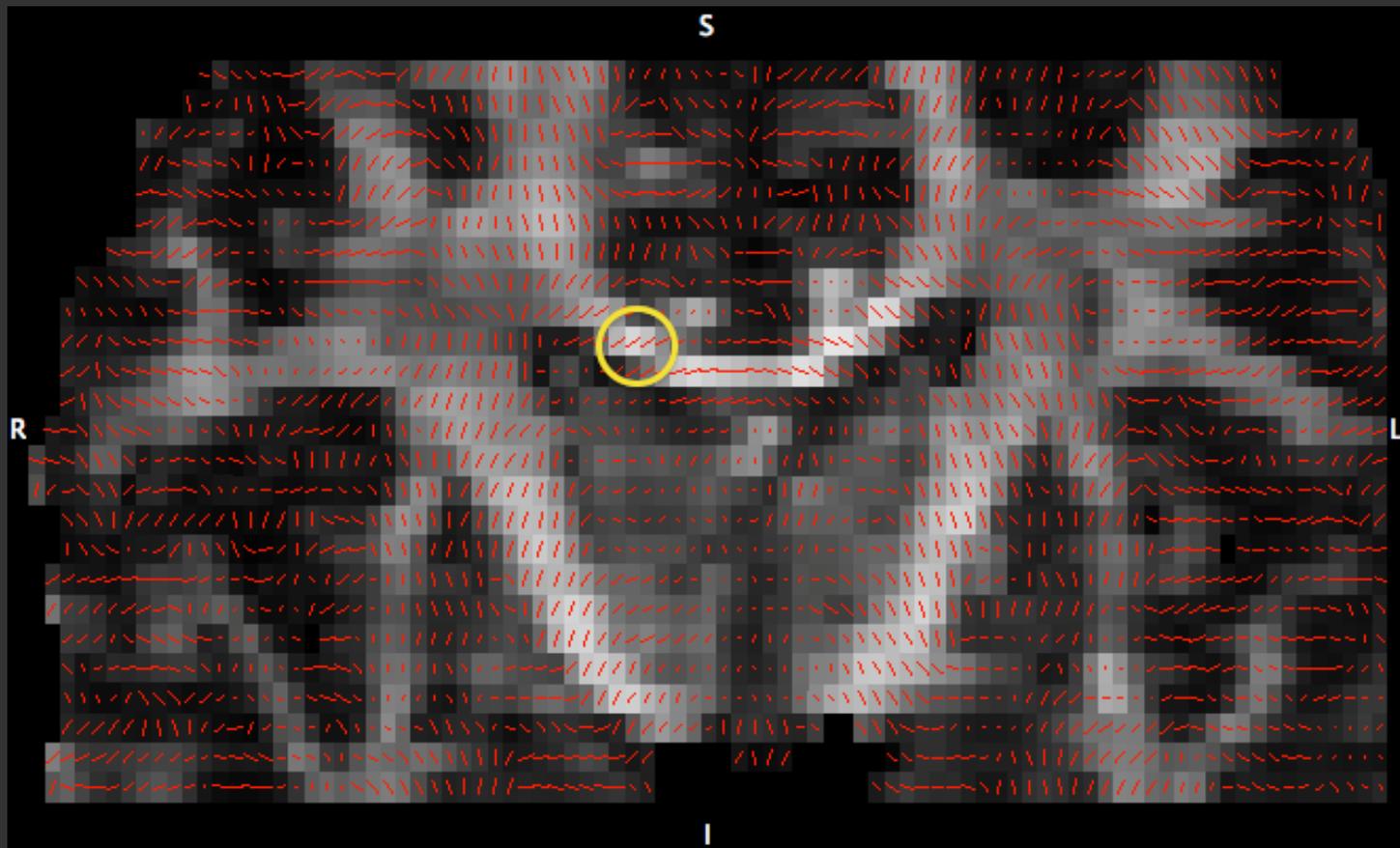
Data analysis steps

- Pre-process images to reduce distortions
 - Either register distorted DW images to an undistorted (non-DW) image
 - Or use information on distortions from separate scans (field map, residual gradients)
- Fit a diffusion model at every voxel
 - DTI, DSI, Q-ball, ...
- Do tractography to reconstruct pathways and/or
- Compute measures of anisotropy/diffusivity and compare them between populations
 - Voxel-based, ROI-based, or tract-based statistical analysis



Caution!

- The FA map or color map is not enough to check if your gradient table is correct - **display the tensor eigenvectors as lines**
- Corpus callosum on a coronal slice, cingulum on a sagittal slice



Tutorial

- Use `dt_recon` to prepare DWI data for a simple voxel-based analysis:
 - Calculate and display FA/MD/... maps
 - Intra-subject registration (individual DWI to individual T1)
 - Inter-subject registration (individual T1 to common template)
 - Use anatomical segmentation (`aparc+aseg`) as a brain mask for DWIs
 - Map all FA/MD/... volumes to common template to perform voxel-based group comparison

